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# Finite Elements in Engineering and Science

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# Invited paper: Effect of nonlinear behaviour of structures under earthquake action

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**Abstract**

In many cases the maximum dynamic response of a nonlinear structure is evaluated using so-called inelastic design spectra. These inelastic spectra are gained dividing the elastic spectra by behaviour factors, which consider the effect of plastic deformations enabled by the ductility of the structure. The present paper reports about comparisons of nonlinear time-history analyses with the application of behaviour factors and inelastic response spectra, respectively. The investigation of typical structures and earthquakes shows the approach by means of inelastic spectra to be very rough. If more reliable informations about forces and displacements of nonlinear structures are required, they have to be obtained by carrying out time-history analyses under consideration of the nonlinear behaviour of the structure.

## 1 Introduction

Common standards like UBC '94 [9], EC 8 [3], API 620 [1] etc. allow to consider the effects of the nonlinear behaviour of a structure, dividing its elastic response - assuming it remains elastically beyond the real yield limit - by a factor which is in a range between 4.0 and 12.0, depending on the statical system and the material. Sometimes not the elastic response of a structure is reduced, but in advance the spectrum, i.e. the loading, itself, however, leading to same results (inelastic design spectrum). The reason, why this approach is permitted, may be the object to avoid a nonlinear dynamic time-history analysis requiring much more time and money than a response spectrum analysis, which is strictly limited to linear systems. An early substantiation of response and design spectra for inelastic systems has been given by Blume/Newmark/Corning [2]. In [5], [6] formulas have been proposed for the determination of an appropriate reduction factor (= behaviour factor), presupposing a given ductility. The question, in how far the effect of nonlinearities can be considered by means of simple reduction of the elastic results, is discussed following by investigation of an uplifting steel tank, which has been treated as a typical nonlinear 1DOF-system with *DIANA*.

## 2 Uplifting Steel Tank as Example of Nonlinear Behaving Structures

A non-anchored storage tank begins to lift up, if the overturning moment due to horizontal excitation exceeds a certain value. Fig. 1 shows a steel tank in the moment of uplifting. Plastic hinges are arising at both sides, the uplifting and the compression side, where the steel shell may buckle additionally. This not only geometrical, but also physical nonlinear behaviour of the structure can be modelled by a bending beam constrained by a nonlinear rotational spring (fig. 2a) [4]. The relation between overturning moment $M_{ot}$ and rotation $\varphi$ is linear as long as the tank does not uplift or if the tank is anchored. However, if the tank lifts up, the rotation $\varphi$ increases rapidly, while the overturning moment $M_{ot}$ reaches a

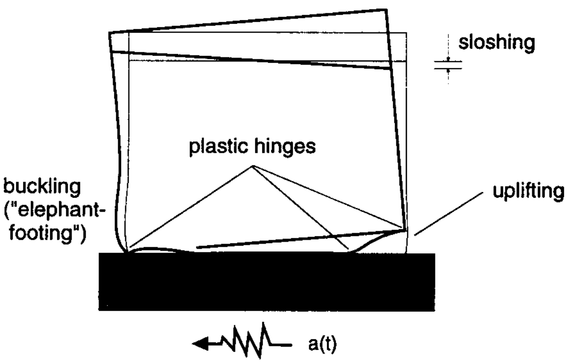


**Fig. 1:** Uplifting steel tank due to earthquake excitation

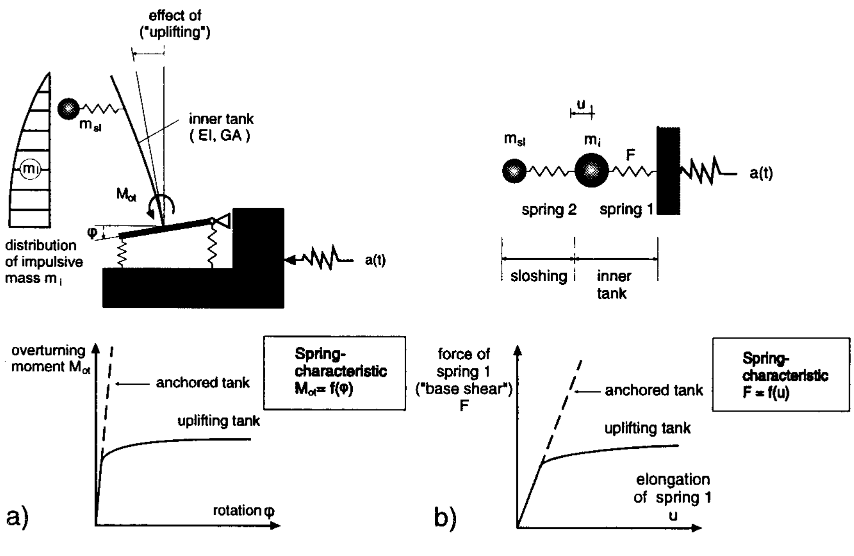


**Fig. 2:** Idealization of uplifting steel tank
a) Cantilevering beam with base rotation $\phi$
b) 2-degree-of-freedom-model

plateau. This model can be simplified by a 2DOF-system (fig. 2b), if the steel tank is represented only by its first fundamental eigenmode. The elastic translational spring constant of the non-uplifting (or anchored) tank can be evaluated acc. to [7]. Similar to the $M_{ot}$-$\varphi$-relation of the model in fig. 2a, also the force-displacement relation (F-u-relation) shows a plateau for the uplifting tank. Because the sloshing of the liquid has not a significant influence on the behaviour of the steel tank, it can be neglected, so that the model turns into the classical nonlinear 1DOF-system, which is the base of the investigations in section 4.

# 3 Proposals for Reduction Factors

A sufficient ductility is an essential requirement for structures in seismic active areas. Ductility means generally, that after reaching the yield stress in one or more parts of the structure the deformation may increase without collapse. The ductility is defined as the relation between the maximum deformation, which can be sustained without loss of structural integrity ($u_u$), and the deformation at the beginning of yield ($u_y$):

$$\mu_\Delta = \frac{u_u}{u_y} \tag{1}$$

By structural detailing it has to be ensured that the structural forces, which are necessary for equilibrium, are kept constant during increase of deformation from $u_y$ to $u_u$.

It is clear that the resistance of a structure against seismic excitation increases with $\mu_\Delta$. Therefore, proposals have been made in [2], [5], [6], to reduce the seismic response resulting from an elastic analysis by so-called reduction factors $\alpha_\mu$, which depend on the ductility $\mu_\Delta$.

In the definition of $\alpha_\mu$ two principles have been proposed:
1) principle of same maximum displacement (fig. 3a) and
2) principle of same work (fig 3b).

The principle of same maximum displacement makes the assumption that a linear-elastic 1DOF-oscillator shows the same maximum displacement $u_u$ as an elasto-plastic one. The reduction factor $\alpha_{\mu 1}$ is calculated as follows:

$$\alpha_{\mu 1} = \frac{F_y}{F_{el}} = \frac{u_y}{u_u} = \frac{1}{\mu_\Delta} \tag{2}$$

The second principle is based on the assumption that the elastic and the elasto-plastic system perform the same work of deformation E:

$$E = \frac{1}{2} \cdot F_{el} \cdot \left( u_y \cdot \frac{F_{el}}{F_y} \right) = F_y \cdot \left( u_u - \frac{u_y}{2} \right)$$

The reduction factor $\alpha_{\mu 2}$ is then:

$$\alpha_{\mu 2} = \frac{F_y}{F_{el}} = \frac{1}{\sqrt{2\mu_\Delta - 1}} \tag{3}$$

Fig. 4 shows both reduction factors depending on the ductility factor $\mu_\Delta$. The greater the ductility factor $\mu_\Delta$, the greater is the difference between both functions (2) and (3). Acc. to [6] $\alpha_{\mu 2}$ (principle of same work) leads to realistic results for frequencies in the range of 2 to 10 Hz, while $\alpha_{\mu 1}$ (principle of same maximum displacement) matches better for low frequencies (< 0.7 Hz). For high frequencies the 1DOF-oscillator is quasi-elastic, so that no reduction is made ($\alpha_\mu = 1.0$). Frequencies inbetween (0.7 Hz < f < 2 Hz, 10 Hz < f < 33 Hz) are interpolated logarithmically.

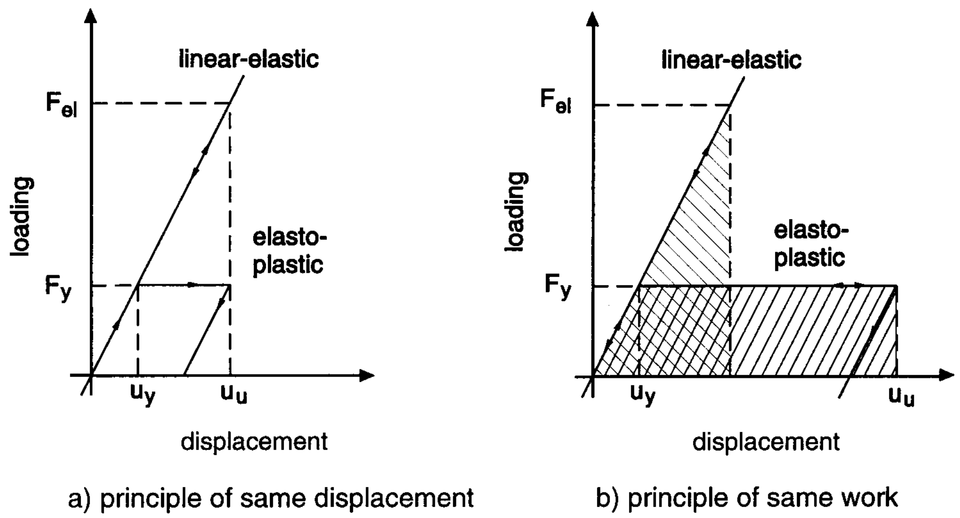


**Fig. 3:** Principles for mathematical evaluation of reduction factors of elasto-plastic 1DOF-oscillators

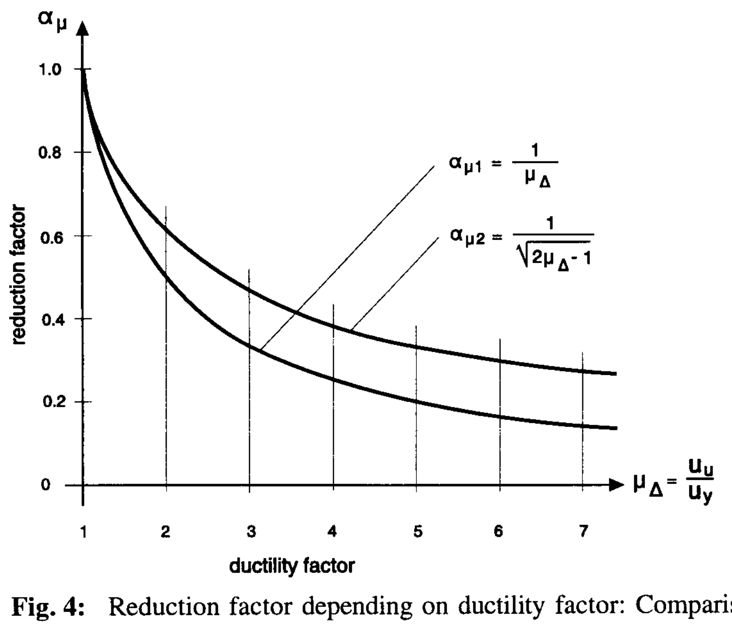


**Fig. 4:** Reduction factor depending on ductility factor: Comparison of $\alpha_{\mu 1}$ (principle of same maximum displacement) with $\alpha_{\mu 2}$ (principle of same work)

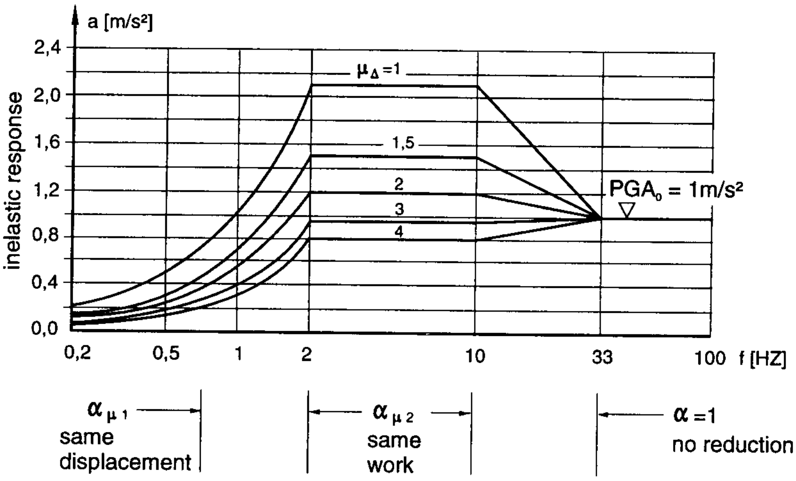


**Fig. 5:** Inelastic design spectrum for a peak ground acceleration PGA = 1.0 $m/s^2$

Dividing the elastic response spectrum by these varying reduction factors ($\alpha_{\mu 1} < \alpha_{\mu 2} < 1$) one obtains a so-called inelastic design spectrum as shown exemplarily in fig. 5.

# 4 Time-History Analyses at Nonlinear 1DOF-system for Checking Mathematical Reduction Factors

For given acceleration time-histories the actual relation between ductility factor and reduction factor can be found by nonlinear analyses. The principle is given in fig. 6. If a system is excitated by a certain earthquake scaled to a peak ground acceleration PGA, which is lower than $PGA_0$, the system remains in the elastic range, i.e. an earthquake scaled to $PGA_0$ leads exactly to the maximum displacement $u_y$; the yield strength is just reached. This $PGA_0$ is different for each time-history. If the PGA is increased beyond $PGA_0$, say 3 times $PGA_0$, the system becomes nonlinear and the maximum occuring displacement $u_u$ is greater than $u_y$. As fig. 6a indicates, the maximum displacement $u_u$ may be different for each considered time-history. A time-history A may lead to a maximum displacement $u_{u,A}$, which is lower than $3 \cdot u_y$ (result of an unlimited linear system), whereas a time-history B may show a greater maximum displacement ($u_{u,B}$). Acc. to equation (1) the ductility factors can be found as:

$$\mu_{\Delta,A} = \frac{u_{u,A}}{u_y}, \quad \mu_{\Delta,B} = \frac{u_{u,B}}{u_y}.$$

Fig. 6b shows the corresponding maximum forces. For earthquakes scaled to PGA $\leq$ $PGA_0$, the maximum force is below or reaches just the yield limit $F_y$. In case of an unlimited elastic system, an earthquake with $3 \cdot PGA_0$ leads to a maximum force of $F_{el} = 3 \cdot F_y$. In case of an elasto-plastic system the maximum force is constant ($F_y$) for all PGA > $PGA_0$. Acc. to the definition of the reduction factor given in section 2, $\alpha_\mu$ can be read from fig. 6b to:

$$\alpha_\mu = \frac{F_y}{F_{el}} = \frac{F_y}{3 \cdot F_y} = \frac{1}{3}.$$

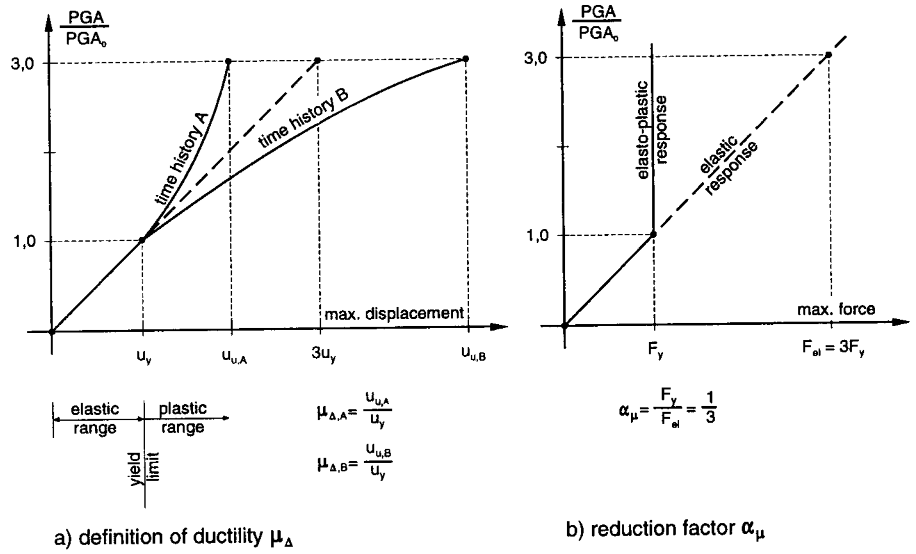


**Fig. 6:** Evaluation of ductility and reduction factors by nonlinear time-history analysis (principle)

In the following the real relation between reduction factor $\alpha_\mu$ and ductility factor $\mu_\Delta$ shall be determined by excitation of a nonlinear 1DOF-oscillator, which describes a liquid storage steel tank as introduced in section 2, using acceleration records of natural earthquakes (fig. 7). The linear spring stiffness $K_0$ for a non-uplifting tank, which remains elastically, is calculated acc. to [7] to $K_0$ = 9660 MN/m. The impulsive part of the liquid - as mentioned above the sloshing part has no significant influence and can be neglected - results acc. to [8] to m = 36 MNs²/m (36 000 to). Therefore, the first natural period of this 1DOF-system is

$$T_0 = 2\pi \cdot \sqrt{\frac{m}{K_0}} = 2\pi \cdot \sqrt{\frac{36}{9660}} = 0.38s\,.$$

This is a typical period of steel storage tanks, which is in the range of maximum amplification of most earthquakes.

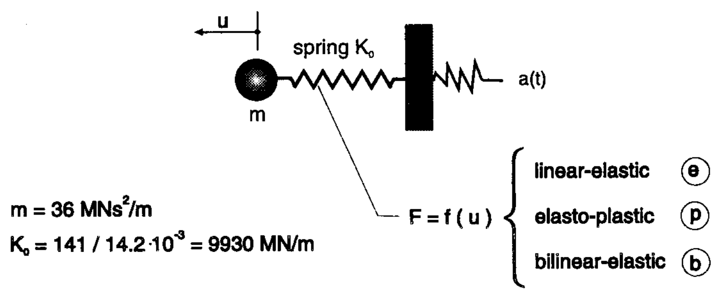


**Fig. 7:** 1DOF-system representing nonlinear behaviour of structure

If the excitation causes uplifting of the tank, the linear spring characteristic ⓔ becomes nonlinear. Acc. to [4] the yield level $F_y$ of the nonlinear spring characteristic can be taken to $F_y$ = 141 MN. This corresponds to a relative displacement of the spring $u_y$ = 14.2 mm. For $u_y$ greater than 14.2 mm the yielding force $F_y$ keeps constant. Because of the complex nonlinear behaviour of the uplifting tank, the spring characteristic is neither exactly a bilinear-elastic function nor exactly an elasto-plastic one, but a hysteresis loop inbetween. However, a strictly bilinear-elastic ⓑ and a strictly elasto-plastic one ⓟ are introduced as a consideration of the possible limits (fig. 8).

This 1DOF-system with different spring characteristics is excitated alternatively by two natural acceleration time-histories a(t):

a) EQ A: NS-component
b) EQ B: EW-component

of the earthquake San Fernando / Lake Hughes (09.02.1971).

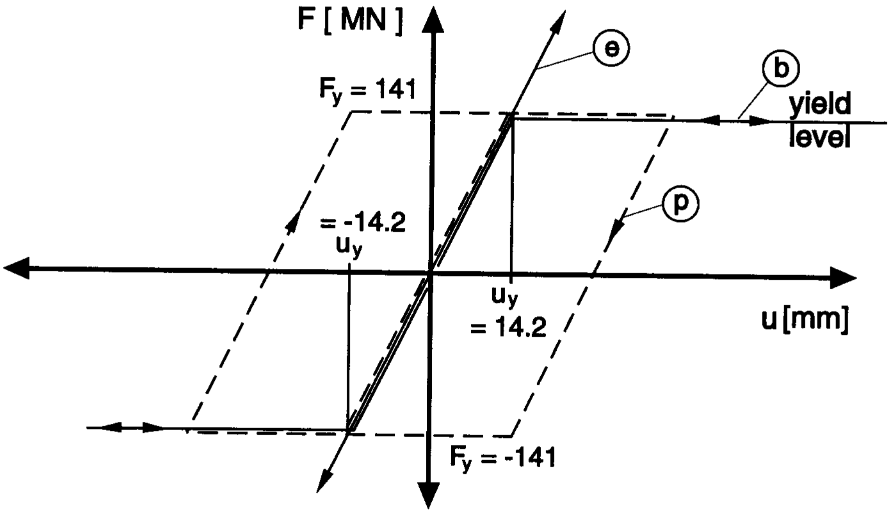


**Fig. 8:** Characteristics for elastic ⓔ, bilinear-elastic ⓑ, and elasto-plastic ⓟ spring

In a first step the 1DOF-system is excitated by the earthquake components, both scaled to a peak ground acceleration $PGA_0$. For EQ A (NS-component) the peak ground acceleration, which leads to $F_{el} = F_y$ was evaluated to $PGA_{0,A}$ = 1.21 m/s², whereas for EQ B (EW-component) $PGA_{0,B}$ results to 3.18 m/s² (for an eigenperiod of 0.38 s).

In a second step the 1DOF-system is loaded again by EQ A and EQ B, however, now scaled to 3 times $PGA_0$ (3.63 m/s² for EQ A and 9.54 m/s² for EQ B). In case of the linear spring characteristic e both time histories lead to the same maximum occuring displacement $u_u = 3 \cdot u_y$ = 42.6 mm, because the calculations are completely linear. In case of the spring characteristics ⓑ or ⓟ the maximum occuring deflection $u_u$ differs from that in the elastic case. The actual maximum displacements are summarized in tab. 1 and fig. 9.

**Tab. 1:** Summary of maximum displacements $u_u$ for earthquakes EQ A and EQ B, both scaled to PGA = $3 \cdot PGA_0$

| | | spring characteristic | | | | |
|---|---|---|---|---|---|---|
| No. | component of San Fernando/Lake Hughes | elastic ⓔ | bilinear-elastic ⓑ | | elasto-plastic ⓟ | |
| EQ A | NS-component | $3 \cdot u_y$ = 42.6 mm | ○ | 27.2 mm | ● | 21.5 mm |
| EQ B | EW-component | $3 \cdot u_y$ = 42.6 mm | □ | 94.5 mm | ■ | 44.0 mm |

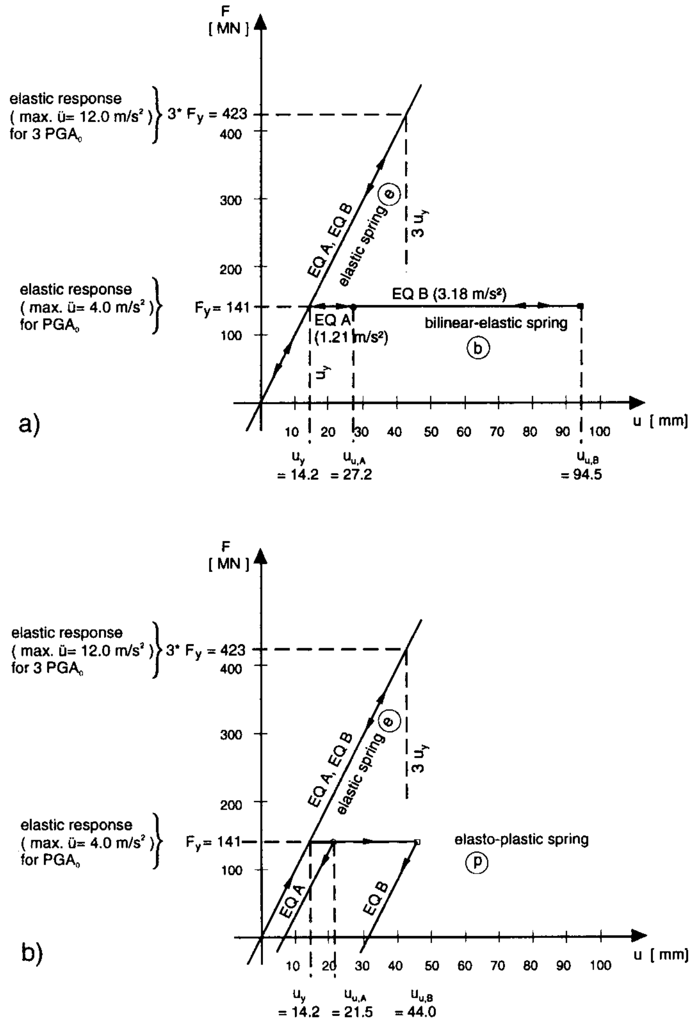


**Fig. 9:** Detailed results of time history analyses
a) dynamic response for elastic and bilinear-elastic spring characteristic
b) dynamic response for elastic and elasto-plastic spring characteristic

These maximum displacements of the nonlinear analyses are now related to $u_y$, to obtain the ductility factors acc. to the definition given in equation (1). They are summarized in tab. 2. The reduction factor is in any case $\alpha_\mu = 1/3$. A comparison with the proposals of section 2 is given in fig. 10. For the elasto-plastic system, the proposal of [6] for $\alpha_{\mu 1}$ fits well for EQ B ($PGA_0 = 3.18$ m/s$^2$) and is far on the safe side for EQ A ($PGA_0 = 1.21$ m/s$^2$). The bilinear-elastic system requires greater ductilities.

**Tab. 2:** Ductility factors $\mu_\Delta$ for a constant reduction factor $\alpha_\mu = 1/3$

| | | spring characteristic | |
|---|---|---|---|
| No. | component of San Fernando/Lake Hughes | bilinear-elastic ⓑ | elasto-plastic ⓟ |
| EQ A | NS-component | 1.92 | 1.51 |
| EQ B | EW-component | 6.65 | 3.10 |

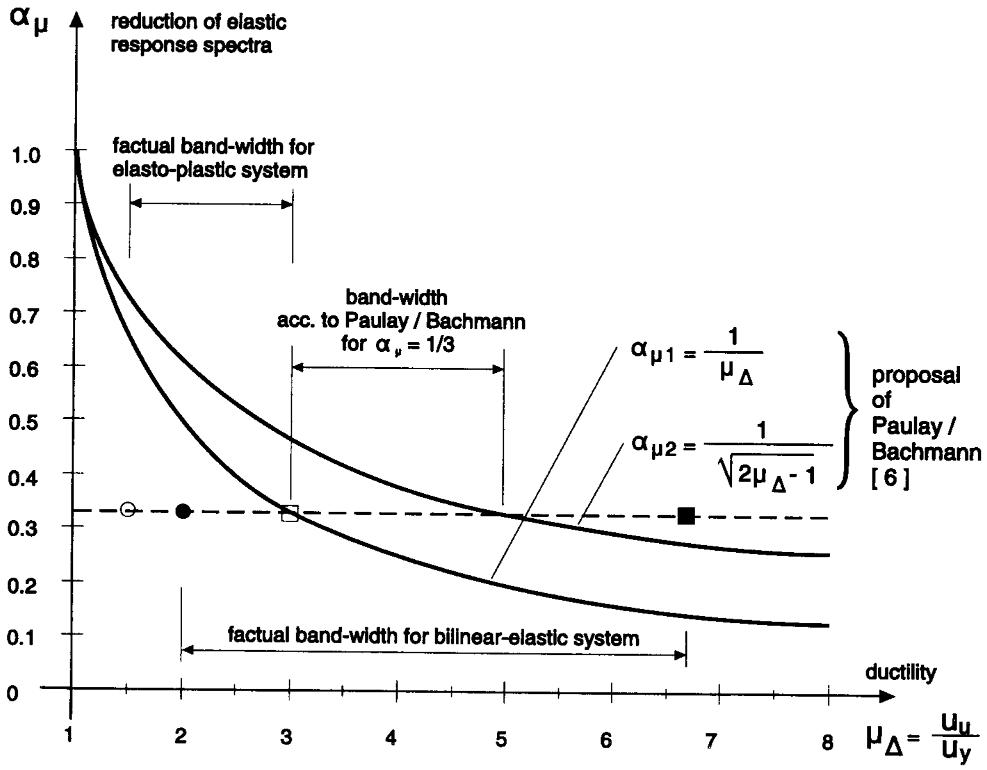


**Fig. 10:** Comparison of results of time-history analyses with mathematical proposal of section 2

## 5 Conclusions

The analyses of nonlinear 1DOF-systems showed, that the ductility factor $\mu_\Delta$ already is scattering considerably even for the two orthogonal directions of one and the same earthquake (NS- and EW-component of the earthquake of San Fernando / Lake Hughes), while the yield level $F_y$ and the frequency of the elastic system have been kept constant. Deviations from the formulas proposed in [5], [6] were to be expected, because they are based on the following arbitrary assumptions:

1) Principle of same maximum displacement: It is not mandatory at all, that the maximum occuring displacement of an unlimited linear system should be the same as that of an elasto-plastic one.

2) Principle of same work: As indicated in fig. 11 there are compared two different types of energy: The reversible energy of the linear system $W_{1,rev}$ with the sum of reversible and dissipative energy $W_{2,rev} + W_{2,dis}$ of the elasto-plastic system.

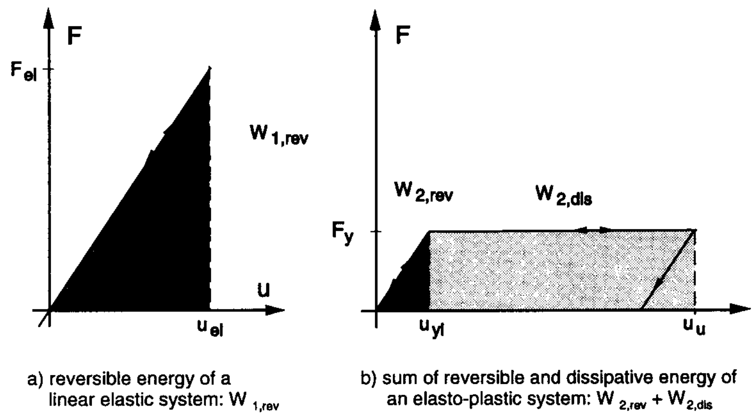


**Fig. 11:** Different types of energy

Elastic design spectra shall constitute an overall cover of the spectra of numerous earthquakes and lead to conservative results for elastic systems. As the seismic response of nonlinear systems varies with the individual time history equally or even more than the response of elastic systems, it is not possible to define "exact" behaviour factors. Therefore, such factors must be far on the safe side. This seems to be the case for the factors proposed in [6], which are provided for the use in the design of multistory buildings. However, behaviour factors in the range of 8 to 12 would presuppose an extreme high ductility of the structure, which usually is not given.

If a more reliable consideration of the nonlinear behaviour of a complex structure is necessary, e.g. for uplifting storage steel tanks, an elastic response analysis with subsequent application of behaviour factors is not sufficient. In this case real nonlinear time-history analyses have to be performed.

# 6 References

[1] API 620: Design and Construction of Large, Welded, Low-Pressure Storage Tanks, Appendix L, 8th edition, June 1990, American Petroleum Institute, Washington.

[2] Blume, J.A., Newmark, N.M., Corning, L.H.: Design of Multistory Reinforced Concrete Buildings for Earthquake Motions. Portland Cement Association, 1961.

[3] Eurocode (EC) 8: Design provisions for earthquake resistance of structures - Part 1-1: General rules - Seismic actions and general requirements for structures, ENV 1998-1-1, May, 1994.

[4a] Malhotra, P.K., Veletsos, A.S.: Beam Model for Base-Uplifting Analysis of Cylindrical Tanks, In: Journal of Structural Engineering, Vol. 120, No. 12, p. 3471-3488, December, 1994.

[4b] Malhotra, P.K., Veletsos, A.S.: Uplifting Analysis of Base Plates in Cylindrical Tanks, In: Journal of Structural Engineering, Vol. 120, No. 12, p. 3489-3505, December, 1994.

[4c] Malhotra, P.K., Veletsos, A.S.: Uplifting Response of Unanchored Liquid-Storage Tanks, In: Journal of Structural Engineering, Vol. 120, No. 12, p. 3525-3547, December, 1994.

[5] Müller, F.P., Keintzel, E.: Erdbebensicherung von Hochbauten, Ernst & Sohn, Berlin, 1984.

[6] Paulay, Th., Bachmann, H., Moser, K.: Erdbebenbemessung von Stahlbetonhochbauten, Birkhäuser Verlag, Basel, 1990.

[7] Sakai, F., Ogawa, H., Isoe, A.: Horizontal, Vertical and Rocking Fluid-Elastic Response and Design of Cylindrical Liquid Storage Tanks, In: Proceedings of the 8th World Conference on Earthquake Engineering (WCEE), San Francisco, 1984.

[8] Seismic Design of Storage Tanks: Recommendation of a Study Group of the New Zealand National Society for Earthquake Engineering, December, 1986.

[9] Uniform Building Code (UBC), 1994, International Conference of Building Officials.